elementary context provided by a simple beam element, it may be readily applied to other, more complex elements. All that is required is to imagine the element supported in a statically determinant manner that corresponds to $s = 0$, determine the internal stress distribution in this configuration under the loads and evaluate the complementary strain energy. The effects will have the appearance of initial element strains. It is believed that this approach fills a theoretical need for an energy-consistent treatment of such loads in the matrix force method.

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Aerodynamic Characteristics of the Slotted Fin

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Nomenclature

A_F = planform area of slotted fin
A^* = planform area of solid fin
$C1$ = induced rolling moment coefficient $L(\phi)/qsd$
$C1_4$ = amplitude of induced rolling moment coefficient
d = missile body diam
$L(\phi)$ = roll torque due to roll angle
$L(\dot{\phi})$ = roll torque due to rolling velocity
P = spin rate
q = dynamic pressure
S = $\pi d^2/4$
V = velocity
δ = fin cant angle

Introduction

THE flight performance of finned bodies is critically dependent on the configurations roll behavior. Because of manufacturing tolerances, slight configurational asymmetries dictate the need for spin to avoid large dispersion. Too low a design spin rate, however, may lead to resonance oscillations,[1] where the trim angle of attack due to asymmetry is amplified to a value inversely proportional to the total damping in the system. Finned configurations near resonance have been observed to develop extremely large angular motions in excess of that predicted by resonance theory alone. This "castastrophic yaw," arising from roll-induced aerodynamic moments causing "roll lock-in" or "lunar motion," was first described by Schneller[2] and later documented during the flight trials of low drag bomb configurations[3]. Magnus instability,[1] generally characterized by large rolling velocities, was noted even earlier by Kent of the Ballistic Research Laboratories.

In 1961, Lugt[4] indicated that slots, or gaps, in the fin planform might radically alter the dynamic angular motion of finned bodies by sweeping away strong wake vortices ordinarily attached to the receding fin at very large angles of attack. Pursuing this possibility, it was shown how the performance of such a basic configuration in free rolling motion responds to fin slots at all angles of attack; it was suggested that these results could be used to alleviate the problem of catastrophic yaw for finned configurations in free flight.[5,6]

These wind-tunnel tests were conducted primarily to demonstrate the feasibility of the slotted fin. Further testing has been performed on a larger sample of slotted fin configurations. It is the purpose of this paper to present a summary of these test results and discuss the effect of slot size on both roll behavior and longitudinal stability.

Wind-tunnel tests

Subsonic wind-tunnel tests were conducted at the Naval Academy to determine the effect of slot size on the longitudinal stability and rolling characteristics of a cruciform finned missile. The tests were conducted at approximately 150 FPS. The test specimen is shown in Fig. 1.

The Naval Academy model had a 3.2 caliber ogive nose with a 4.4 caliber cylindrical afterbody. The model's maximum body diam. was 1.5 in. The fins were rectangular and trapezoidal, with an exposed semispan of 1 caliber.

A free rolling test was conducted to determine the effect of fin slots and fin cant on roll lock-in and roll speed-up.

Figure 2 gives the steady-state spin rate vs angle of attack for the Naval Academy model with solid rectangular fins and approximately no fin cant.

Lock-in exists from 20° to 50° angle of attack. Considerable speed-up exists above 50°. Dual modes of motion exist throughout. Very slow clockwise motion existed below 20° and was not recorded. The fin cant was then varied from zero to a maximum of 8° in order to overcome the lock-in.

Figure 3 gives the steady-state spin rate vs angle of attack for the Naval Academy model with solid rectangular fins and 8° of fin cant. Lock-in now occurs at 30° rather than at 20° which is generally beneficial. The speed-up is hardly affected. Not only is the spin high at high angles of attack but it is also independent of the fin cant.

These results are typical for all solid fins which have been tested. The motion of the Naval Academy model was then investigated with varying slot size and fin cant.

The addition of the slot eliminates roll speed-up if the slot is sufficiently large. Figure 4 gives the motion for the minimum slot size which alleviated roll speed-up for the rectangular fin configuration. Fin cant is nearly zero.

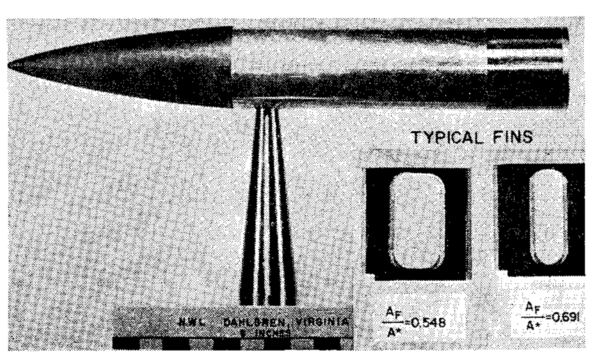

Fig. 1 Wind-tunnel model-Naval Academy.

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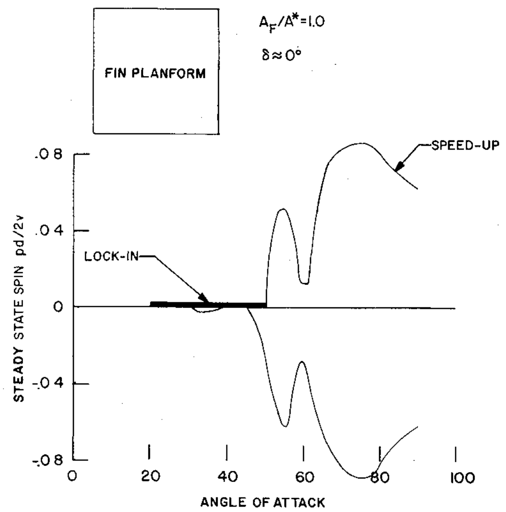

Fig. 2 Steady-state rolling velocity vs angle of attack for Naval Academy model with solid rectangular fins. Fin cant 0.

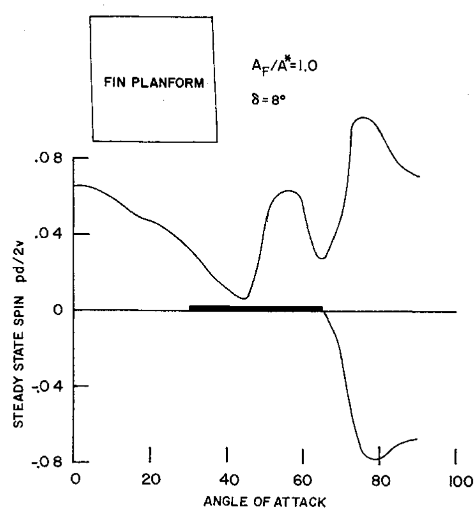

Fig. 3 Steady-state rolling velocity vs angle of attack for Naval Academy model with solid rectangular fins. Fin cant 8°.

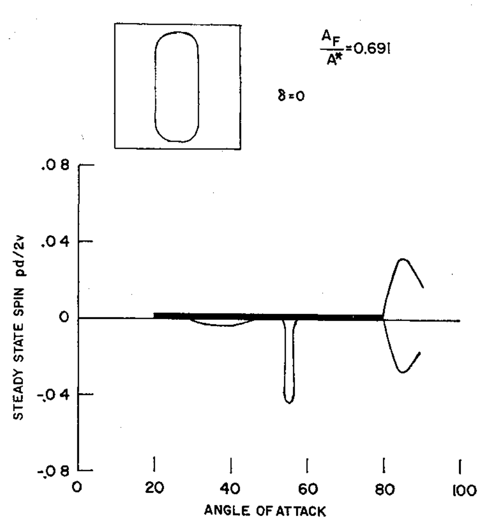

Fig. 4 Steady-state rolling velocity vs angle of attack for Naval Academy model with rectangular fins and a slot. $A_F/A^* = 0.691$.

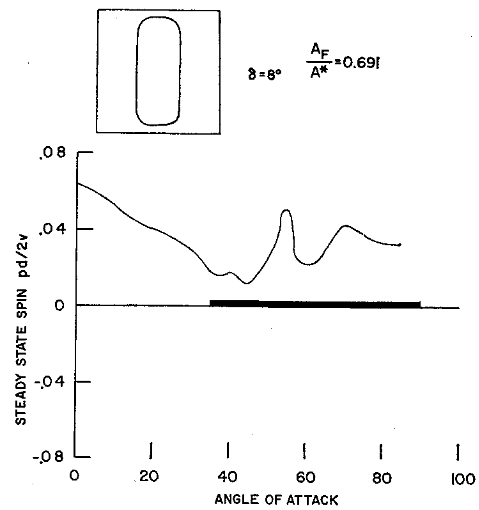

Fig. 5 Steady-state rolling velocity vs angle of attack for Naval Academy model with rectangular fins and a small slot. $A_F/A^* = 0691$.

Some slight residue of speed-up exists. However with the addition of cant as shown in Fig. 5, no speed-up is apparent. The spin is higher at low angles of attack. However it is only $\frac{3}{4}$ of the maximum spin of an uncanted solid fin and the maximum spin of the slotted fin occurs at the smallest angle of attack where it is least critical. The minimum lock-in angle is now moved up to 35°. There is still the possibility of lock-in but only if the missile's rolling motion is stopped. It should also be noted that the roll damping for this configuration is stabilizing, $L(\dot{\phi}) < 0$ for all positive values of $\dot{\phi}$. Above an angle of attack of 35° the missile will spin. If we reduce its spin below a critical value, it will damp. This phenomenon can only occur if the roll damping torque is negative for positive spin rates. Conversely, roll speed-up[7] of the solid fin configuration can only occur if the roll damping is destabilizing.

It was obvious that the ratio of induced rolling moment to fin cant moment was improved. From the response of the model, it was felt that even a slight change in the ratio might eliminate the lock-in mode. Possibly a higher fin cant or more efficient slot shape could have eliminated it. A larger fin cant could not be tested due to the limits of the model design.

Figure 6 shows that a slightly smaller slot nearly eliminates the lock-in mode. However, speed-up is present, even though considerably weakened.

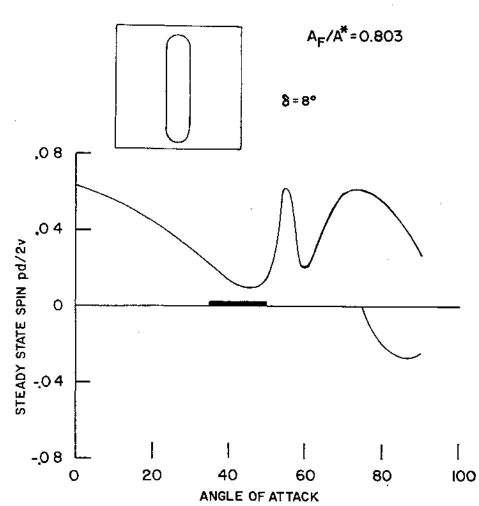

Fig. 6 Steady-state rolling velocity vs angle of attack for Naval Academy model with rectangular fins and a small slot. $A_F/A^* = 0.803$.

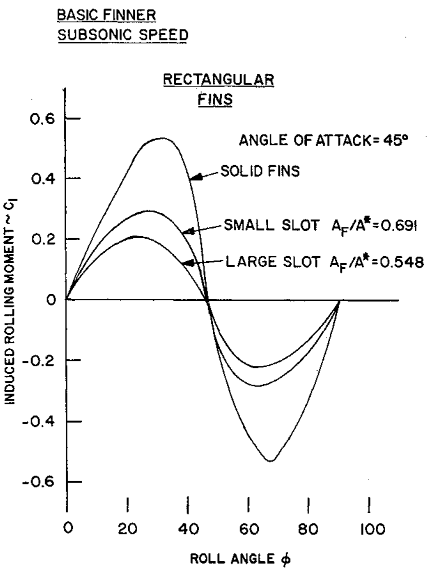

Fig. 7 Typical effect of slot on induced rolling moment.

During the Naval Academy tests, both rectangular and trapezoidal fins were studied. The results obtained were essentially the same for both types of fins.

Single-degree-of-freedom, free oscillation tests were then conducted at the Naval Academy to determine the effect on the missile's longitudinal stability due to slot size.[8] Pitching motion was recorded and these data were fit using a nonlinear, least squares technique. Both the linear and nonlinear contributions of the restoring moment and pitch damping moment were determined. The results of this study indicated that the slot reduces longitudinal stability at low angles of attack but increases it at high angles of attack. Only with extreme slot size ($AF/A^* \leq 0.347$) was the model statically unstable. No dynamic instability was present.

Returning to Fig. 4, one might conclude that the slot itself, without the presence of fin cant, actually promotes lock-in. Returning to Fig. 5, we note that this is not the case at the moderate angles of attack because the minimum lock-in angle is greater.

In order to determine if the slot promoted lock-in at higher angles of attack, a test was conducted at NSRDC to study the effect of slot size on the induced rolling moment. The basic finner[7] (a well-known research configuration) was used as the test specimen because of its availability. Fins identical to those tested at the Naval Academy were studied.

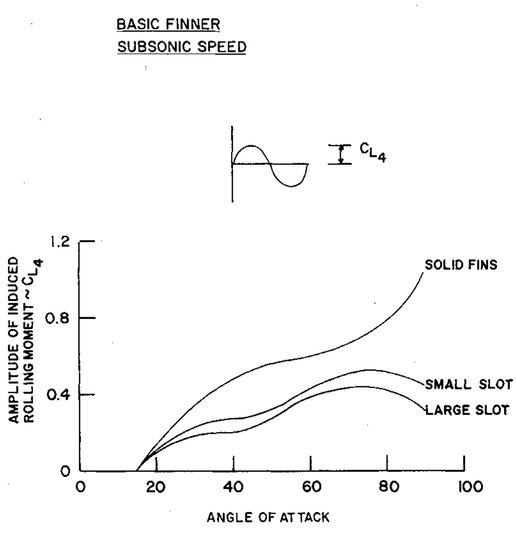

Fig. 8 Effect of slot on amplitude of induced rolling moment.

An internal strain gage balance was used to measure the induced rolling moment. The two planforms for the slot sizes shown are presented in Fig. 1. At an angle of attack of 45° (Fig. 7), the slot significantly reduces the induced rolling moment. The effect of slot size on the induced rolling moment at higher angles of attack is equally dramatic. Figure 8 summarizes the results of the test. It is noted that the induced rolling moment was reduced by as much as 70 percent for the slot sizes tested and the small slot is nearly as efficient as the large slot in reducing the induced rolling moment.

Conclusion

Based on the results of this study it is concluded that, at subsonic speeds, the slotted fin is superior to the solid fin in that it eliminates roll speed-up, appreciably reduced the induced rolling moment, and increases longitudinal stability at high angles of attack. Stability is reduced at low angles of attack. However, the possibility of catastrophic yaw is minimized.

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V-Wings and Diamond Ring-Wings of Minimum Induced Drag

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THE problems solved here should be found in the literature of the classical era of aerodynamics, and it is hard to believe that they could have been overlooked for so many decades; but it appears that the solutions have never been published. The solutions do contribute to some modern interest in nonplanar wings;[1] also the added-mass coefficients

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